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Book Review: Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields

Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.

John Guckenheimer and Philip Holmes

One important aspect of dynamical systems is the study of the long-term behavior of a set of ordinary differential equations (ODEs). In recent years many systems that are simple to write down have been discovered whose solutions are chaotic. They oscillate irregularly, never settling down to a regular pattern. Two trajectories which start close together will separate quickly. Systems whose time evolution is governed by a parameter p can undergo intriguing variations in the behavior of trajectories. In many cases, there are values p^* such that the long-term behavior of typical trajectories of $p < p^*$ is much different than for $p > p^*$. For example, the system may go from stable periodic behavior for $p < p^*$ to chaos for $p > p^*$. Such sudden, discontinuous changes or "bifurcations" are quite common.

Research in chaos and bifurcations in dynamical processes has advanced at a rapid pace during the past decade, acquiring an extraordinary breadth of applications in fields as diverse as fluid mechanics, electrical engineering and neurophysiology. The new results interest a wide spectrum of the scientific community, many of whose members, however, lack the mathematical background necessary to decipher the literature. Accordingly, Guckenheimer and Holmes have written their book as a "user's guide" to an extensive and rapidly growing field.

The book surveys the theory and techniques needed to understand chaotic behavior of ODEs. The first chapter contains a brief introduction of the theory of ODEs; it is a review of topics usually found in a standard text like Hirsch and Smale.[1] The second chapter considers four examples of chaotic systems: the forced van der Pol oscillator, Duffing's equation, the celebrated Lorenz equations, and Holmes' "bouncing ball map" (perhaps more familiar as the map which describes the motion of a periodically forced, damped planar pendulum in the absence of gravity). These examples

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illustrate the ideas discussed throughout the text and thereby unify the presentation. Approximately three of the seven chapters are devoted to bifurcation theory; most of this material treats the bifurcations of fixed points and periodic orbits. Another chapter has a nice discussion of the Smale horseshoe, symbolic dynamics, and their relation to so-called "strange attractors" and chaos.

Guckenheimer and Holmes have undertaken a difficult task; they are faced with the problem of how to introduce a diverse audience to the field, yet keeping the book to a reasonable size. It is necessary to exclude soliton theory, and little attention is given to one-dimensional maps and Hamiltonian systems, but these are appropriate omissions. We applaud their selection of typics, which gives balanced coverage to a variety of subjects.

The book is rewarding reading, but it is also quite difficult. The elementary chapters are suitable for an introductory graduate course for mathematicians and physicists, but the instructor is cautioned that students without an advanced undergraduate mathematics background will not understand much of the material. Indeed, the instructor should be sure he can read the book himself before selecting it for a course. Its excellent survey of the mathematical literature makes it a valuable reference.

1. Morris W. Hirsch and Stephen Smale, *Differential Equations, Dynamical Systems, and Linear Algebra* (Academic Press, New York, 1974).

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